Practice Paper 2015-16

Subject – Mathematics

Time: 3 hrs **M. M: 100**

General Instructions:

1. **All questions are compulsory.**
2. **Please check that this question paper contains 26 questions.**
3. **Question 1 – 6 in Section A are very short – answer type questions carrying 1 mark each.**
4. **Questions 7 – 19 in Section B are long – answer I type question carrying 4 marks each.**
5. **Questions 20 – 26 in Section C are long – answer II type question carrying 6 marks each.**
6. **Please write down the correct serial number of the question before attempting it.**

Section – A

Q. 1 Let $f(x) = [x]\ and\ g(x) = |x|,\ find\ (gof)\left(\frac{-5}{3}\right) - (fog)\left(\frac{-5}{3}\right)$.

Q. 2 Evaluate tan $\left[2\ tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right]$.

Q. 3 If A is an invertible matrix of order 3 and |A| = 5, then find the value of $|\ A^{-1}\ |$

Q. 4 If $\vec{a}\ and\ \vec{b}$ are the two vectors such that $\left|\vec{a}\ .\ \vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$, write the angle between $\vec{a}$ and $\vec{b}$.

Q. 5 Find degree of the differential equation $\mathrm{y = x\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dx}{dy}\right)^2}}$.

Q. 6 Write the equation of a plane when foot of perpendicular from origin to the plane is (2, 1, 1).

Section – B

Q. 7 Show that the relation R on the set
A=$\{\mathrm{x} \in \mathrm{Z}: 0 \le \mathrm{x} \le 12\}$, given by $\mathrm{R} = \{(\mathrm{a,b}): |\mathrm{a-b}|$ is $a\ multiple\ of\ 4\}$ is an equivalence relation.
Find the set of all elements related to 1.

OR

Let * be a binary operation on the set $\{0, 1, 2, 3, 4, 5\}$ defined as a∗b = $\begin{cases} a+b & if \quad a+b < 6 \\ a+b-6 & if \quad a+b \ge 6 \end{cases}$
Find the identity elements and the inverse elements of each element of the set for the operation *.

Q. 8 Prove $2\ tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right) = cos^{-1}\left(\frac{a\ cos\theta\ +b}{a+b\ \cos\ \theta}\right)$.

OR

Find the greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$.

Q. 9 Using properties of determinants. $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (\mathrm{a+b+c})(\mathrm{a^2+b^2+c^2})$

OR

Prove that $\begin{vmatrix} a+bx^2 & c+dx^2 & P+qx^2 \\ ax^2+b & cx^2+d & Px^2+q \\ u & v & w \end{vmatrix} = (x^4-1)\begin{vmatrix} b & d & q \\ a & c & P \\ u & v & w \end{vmatrix}$

Q. 10 Evaluate $\int_{-2}^{2} |\mathbf{x\ Cos\ \pi x}|$.

Q. 11 If $y = b\tan^{-1}\left(\frac{x}{a} + \tan^{-1}\frac{y}{x}\right)$, then prove $\frac{dy}{dx} = \frac{b}{a}\left[\frac{x^2+y^2-ay}{(x^2+y^2)\,\text{Sec}^2\,\frac{y}{b} - bx}\right]$.

OR

If $\mathbf{f(x) = Cot^{-1}}\left(\frac{\mathbf{x^x - x^{-x}}}{\mathbf{2}}\right)$ then prove that the value of $f'(1) = -1$

Q. 12 Discuss the differentiability of $f(x) = |x-1| + |x-2|$.

Q. 13 Evaluate $\int_0^4 (3x^2 + e^{2x})\,dx$, as a limit of sums.

Q. 14 Evaluate $\int \frac{\text{Sin } x}{\text{Sin } 4x}\,dx$

OR

Evaluate : $\int (2Sin\,2x - Cos\,x)\sqrt{6 - Cos^2x - 4\,Sinx}$ dx.

Q. 15 A is known to speak truth 3 times out of 5 times. He throws a die and reports that it is 1. Find the probability that is was actually 5?

The person who always speak the truth have better social recognition, explain.

OR

In a game Neha wins a rupee for a six and loses a rupee for any other number. Neha has been asked to throw the dice thrice but she has to quit when he get six. Find her expected earnings.

Explain the importance of awards for players.

Q. 16 If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{\gamma} = \hat{\jmath} - \hat{k}$,find the vector $\vec{\beta}$ such that $\vec{\alpha} \times \vec{\beta} = \vec{\gamma}$ and $\vec{\alpha}.\vec{\beta} = 3$

Q. 17 Find the coordinates of point on line $\frac{x-1}{2} = \frac{y+2}{2} = \frac{z-3}{6}$,which are at a distance of 3 units from the point (1, -2, 3).

Q. 18 If the straight line $x\text{Cos}\theta + y\text{Sin}\theta = p$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $a^2\text{Cos}^2\theta + b^2\text{Sin}^2\theta = p^2$.

OR

Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

Q.19 Separate the interval $\left[0.\frac{\pi}{2}\right]$ into sub intervals in which $f(x) = Sin^4x + Cos^4x$ is increasing or decreasing.

Section – C

Q. 20 Two schools A and B want to award their selected teachers on the values of honesty, hardwork and regularity. The school A wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to 3,2 and 1 teachers with a total award money of Rs. 1.28 lakhs. School B wants to spend Rs. 1.54 lakhs to award its 4, 1 and 3 teachers on the respective values (by giving the same award money for the three values as before). If the total amount of award for one prize on each value is Rs. 57000, using matrices, find the award money for each value.

Q. 21 Prove that $\int_{-\pi}^{\pi} \frac{2x(1+Sin\,x)}{1+Cos^2\,x}\,dx = \pi^2$

OR

Evaluate: $\int_0^1 x\,(\tan^{-1}x)^2\,dx$.

Q. 22 Find the equation of the plane passing through the point P $(1, 1, 1)$and containing the lines

$\vec{r} = \left(-3\hat{\imath} + \hat{\jmath} + 5\hat{k}\right) + \lambda\,(3\hat{\imath} - \hat{\jmath} - 5\hat{k})$. Also show that the plane contains the line $\vec{r} = (-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) + \mu\,(\hat{\imath} - 2\hat{\jmath} - 5\hat{k})$.

OR

A plane meets the x,y and z axes at A,B and C respectively, such that the centroid of the triangle ABC is (1,-2,3). Find the Vector and Cartesian equation of the plane.

Q. 23 If a young man rides his motor cycle at 25 km/hr, he has to spend Rs.2 per km. on petrol, if he rides it at a faster speed of 40 km/hr, the petrol cost increases to Rs. 5 per km. He has Rs.100 to spend on petrol and wishes to find what is the maximum distance, he can travel within one hour. Express this as a linear programming problem and then solve it.

How important do you think it is to use public transportation?

Q. 24 A given quantity of metal is to be cast into a half cylinder with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its semi-circular ends is π : (π + 2).

OR

Find area of the greatest isosceles triangle that can be inscribed in a given ellipse having its vertex coincident with one end of the major axis.

Q. 25 Sketch the region bounded by the curves $\left\{(x, y): |x - 1| \leq y \leq \sqrt{5 - x^2}\right\}$ and find its area using integration.

Q. 26 A bag contains 25 balls of which 10 are purple and the remaining are pink. A ball is drawn at random, its colour is noted and it is replaced, 6 balls are drawn in this way, find the probability that

(i) all balls were purple
(ii) not more than 2 were pink
(iii) an equal number of purple and pink balls were drawn
(iv) atleast one ball was pink.

Explain the meaning of three colours of our national flags.

OR

Two unbiased dice are thrown, find the probability that the sum of the numbers obtained on the two dice is neither a multiple of 2 nor multiple of 3.

Submitted By- Pankaj Bisht
(P.G.T. MATHEMATICS)
Contact No. - 9711116540